WHAT IS A CONGRESSIONAL SEAT WORTH? ESTIMATING THE VALUE USING AUCTION THEORY

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Abstract. This paper models campaign expenditures as bids in an asymmetric all-pay auction. It predicts intuitive mixed Nash expenditure strategies and electoral outcome probabilities. As district demographics, such as partisan voter registration statistics, favor a candidate more heavily, the mixed expenditure strategies converge pointwise to the classic Median Voter Model. The parameter of the model is then estimated based on the predicted moments of the expenditure distribution. We use the district level voter registration statistics and observed campaign expenditure behavior to estimate the value of a Congressional House seat to be about $4.5 million.

1. Introduction

What is the the optimal amount to spend on a Congressional election? While one common answer is "As much as possible", campaign expenditure filings shows that candidates generally do not expend their full budget on an election. Another common answer would be "More than the other candidate." However, the candidates do not know the amount that there opponent is going to spend. Additionally, what is a Congressional seat worth? It seems obvious that the answer to this question should help determine the optimal amount to spend on an election.

To answer these questions, we model the candidate expenditure decision as a bid in a first-price asymmetric all-pay auction, solve for the mixed Nash expenditure strategies, and estimate the value of a Congressional House seat based on the observed campaign expenditure behavior. We model the campaign expenditure game as an asymmetric all-pay auction for three reasons. First, it is an auction because electoral outcomes are a discrete set. Second, since losing candidates do not recover their bids (campaign expenditures), the auction is all-pay. Finally, it is asymmetric as district demographics may favor one candidate. The model predicts mixed Nash expenditure strategies that are highly intuitive and yield realistic electoral outcome

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probabilities. In the limit, as the fraction of voters who respond to expenditures diminishes, the model returns the classic Median Voter Model.

Other work that models the optimal amount of campaign expenditure include Prat (2002b). Prat (2002b) develops a structural model of campaign spending in which advertising sways pivotal voters. Like previous models, he assumes that candidates always exhaust their entire budget. As observed in the data, candidates often spend much less or much more than their official budget. In this paper, we seek to explain expenditure behavior without assuming that candidates always expend exactly their budget. To do so, we model expenditures as bids in an all-pay auction.

This paper utilizes previously derived auction results to model the campaign expenditures as bids. Amann & Leininger (1996) solve generic mixed Nash equilibrium strategies for asymmetric all-pay auctions. Che & Gale (1998) apply the equilibrium analysis to a lobbying setting and show how the lobbying expenditures would change under different levels of a campaign contributions cap. Neither of these paper use the model predictions to estimate the parameters of the bids distributions. Hendricks & Porter (1988) empirically analyze asymmetric mixed Nash bid strategies by oil companies on federal off-shore tracts. Their paper uses the ex-post identification of the object value to test whether bid behavior was competitive or collusive. In this paper, we use the observed expenditure behavior to estimate the object valuation, the per partisan valuation of a Congressional House seat.

The contribution of this paper is to produce an empirically testable model in which candidates do not necessarily expend their entire budgets and which accounts for large variation in expenditure behavior between congressional districts with similar demographics. As far as the authors are aware, this is the first paper to empirically estimate an auction model when the underlying data is not produced from an actual auction. We differ from previous empirical auction work by showing that we can estimate the parameters using the moments of the predicted distributions instead of using maximum likelihood techniques. We use the first two predicted moments of the optimal expenditure distributions to estimate candidates' valuation of a Congressional House seat. We estimate a Congressional house seat to be worth about $4.5 million.[1]

The organization for the remainder of this paper is as follows: Section 2 explains the model, derives the equilibrium, and explains its intuition. Section 3 explicitly outlines the testable predictions of the model. Section 4 describes our the data and the restrictions used to estimate the value of the congressional seat. Section 5 states the results and provides qualitative analysis, and Section 6 concludes.

[1]All dollar amounts are in 2008 terms.

2. Finance Augmented Median Voter Model (FAMVM)

In this model, campaign expenditures earn candidates a fraction of the advertising-responsive voters. Modeling expenditures in this fashion implies that candidates' expenditures are bids in an asymmetric all-pay auction where the object of valuation is the elected office. This is appropriate because in an election, the set of outcomes is a discrete set. Further, we use the all-pay auction framework since the losing candidates' expenditures are never recovered. Last, we allow asymmetry in the model to accommodate districts where candidates have an advantage. We assume that the candidates moves are simultaneous and information is complete.

The players are the a Congressional district's two campaigning candidates, one Democratic candidate denoted with subscript D and one Republican candidate denoted with subscript R. Each candidate simultaneously chooses a level of campaign expenditure e_D and e_R. In our model, candidates can be assured of the support of their base of b_p voters (for $p = D, R$). Candidates behave as partisan social planners, attempting to maximize the payoffs to their base. There is a common per partisan value (v) of the congressional seat. Therefore if she wins, a candidate's payoff consists of the total value of the seat to their base ($b_p v$), minus their expenditures. If they lose the election their payoff is just equal to the negative of their expenditures.

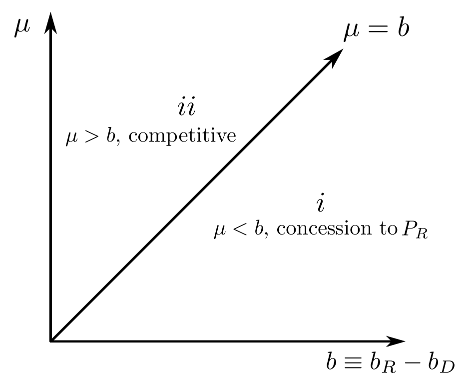
A candidate gets elected to office if they accumulate more votes than their opponent. Auction asymmetries enter in that each candidate may have a different mass of resolute voters behind them, $b_D \neq b_R$. Swing voters, denoted μ, respond to the relative amounts of candidate expenditures (e.g. advertising). We assume the simplest case of a linear proportional advertising technology. So, for example candidate P_D's total number of votes received is $b_D + \mu\frac{e_D}{e_D+e_R}$. Table 1 summarizes the set up of this auction.

Table 1. Notation Summary

Candidates	P_D	P_R
Resolute Voters	b_D	b_R
Expenditures	e_D	e_R
Total Votes	$b_D + \mu\frac{e_D}{e_D+e_R}$	$b_R + \mu\frac{e_R}{e_D+e_R}$
Objective Function	$\max[b_D v * Prob(win_D \mid e_D) - e_D]$	$\max[b_R v * Prob(win_R \mid e_R) - e_R]$

Given a district environment (b_D, b_R, μ) and a v value, the model predicts candidate expenditures. The optimal expenditure strategy is a probability distribution over a continuum of expenditure levels. From here on out we assume without loss of generality that the Republican candidate has a base advantage ($b_R > b_D$) and is referred to as the advantaged candidate. The Democratic candidate is the disadvantaged candidate. For notational simplicity, define $b \equiv b_R - b_D$ as her net base advantage.

FIGURE 2.1. The District State Space



First, consider the simplest case, when the base advantage is larger than the mass of advertising responsive voters. Call this case i. Even if the disadvantaged candidate wins all swing mass, he would still lose.

Theorem 1. *If $\mu < b$, in the unique Nash equilibrium neither candidate expends, and therefore the advantaged candidate will win by concession.*

Proof. There is no benefit to expenditures by either candidate, as P_R will always win.

$$b_R - b_D > \mu \Rightarrow b_R > b_D + \mu$$

Therefore, any expenditures by either player only decreases that player's expected payoff. □

Now, we consider the slightly more complicated, but more interesting case: when the mass of advertising responsive voters is potentially pivotal. Now $\mu > b$. This is region ii on the state space graph, Figure 2.1. Define $s \equiv \frac{\mu - b}{\mu + b}$ and $\gamma \equiv \frac{b_D}{b_R}$. Before continuing we prove the following two propositions:

Proposition 2. *The disadvantaged candidate expends no more than their valuation $b_D v$.*

Proof. Spending more than her valuation is strictly dominated by spending zero. The former always results in a negative payoff, whereas the later must result in a non-negative payoff. □

Proposition 3. *The advantaged candidate never spends more than $b_D vs$.*

Proof. Given 2, $e_r = b_D vs$ will result in P_R tying with P_D. Thus, for any $\epsilon > 0$, spending $b_D vs + \epsilon$ or $b_D vs + \frac{\epsilon}{2}$ both provide a guaranteed electoral victory, but the later does so at a lower cost. In other words, all expenditures strictly greater than $b_D vs$ are strictly dominated. □

Theorem 4. *If $\mu > b$, the following pair of CDFs is a mixed Nash equilibrium to the game:*

$$G_D(e_D) = \begin{cases} 0 & \text{if } e_D < 0 \\ 1 - \gamma s + \frac{e_D s}{b_R v} & \text{if } e_D \in [0, b_D v] \\ 1 & \text{if } e_D > b_D v \end{cases} \tag{2.1}$$

$$G_R(e_R) = \begin{cases} 0 & \text{if } e_R < 0 \\ \frac{e_R}{b_D v s} & \text{if } e_R \in [0, b_D v s] \\ 1 & \text{if } e_R > b_D v s \end{cases} \tag{2.2}$$

Proof. Let each party's's expenditure CDF be denoted by G_p. The candidate who earns more total votes wins the election. Simple algebra shows that P_D wins if $e_D s > e_R$, and similarly for P_R. Thus the maximization problems faced by the players are:

$$P_D : \max_{e_D} \quad b_D v G_R(e_D s) - e_D \tag{2.3}$$

$$P_R : \max_{e_R} \quad b_R v G_D(\frac{e_R}{s}) - e_R \tag{2.4}$$

In equilibrium each player must receive a constant expected payoff (denoted k_p) from any level of expenditure in their support. Combining this with a change of variables yields:

$$G_D(e_D) = \frac{k_R + e_D s}{b_R v} \tag{2.5}$$

$$G_R(e_R) = \frac{k_D + \frac{e_D}{s}}{b_D v} \tag{2.6}$$

Proposition 2 allows us to pin down k_R by setting $G_D(b_D v) = 1$. Plugging this into Equation 2.5 and solving gives us

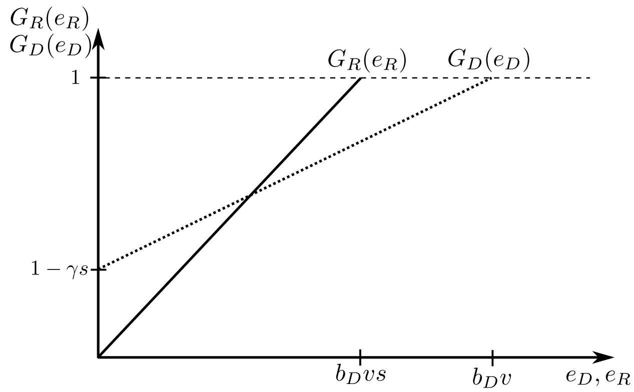
$$k_R = (b_R - b_D s)v \tag{2.7}$$

Similarly, Proposition 3 and Equation 2.6 allow us to do the same for k_D:

$$k_D = 0 \tag{2.8}$$

Plugging the above constants in to Equations 2.5 and 2.6 proves the theorem. □

FIGURE 2.2. Strategy CDFs



By modeling campaign expenditures as bids in a first-price asymmetric all-pay auction, we have solved candidates' optimal campaign expenditure strategies for both candidates as mixed Nash equilibrium best-responses to each other. In region i, the optimal strategy by the candidates is trivial (both candidates always expend 0). In region ii, the optimal expenditure strategy is to play a distribution over possible expenditures. The distribution depends on the district's demographics, specifically, the particular location in the (b, μ) state space. The disadvantaged candidate spends 0 (concedes) with positive probability, and mixes uniformly between expenditures up to the office valuation the remainder of the time. The advantaged candidate's optimal strategy is to expend according to a uniform distribution over all expenditure levels up to a fraction of their valuation.

2.1. **Electoral Outcome Probabilities.** The model's predicted expenditure distributions imply electoral outcome predictions. We use the derived strategies to calculate the probabilities of an advantaged and a disadvantaged win, respectively (in region ii).

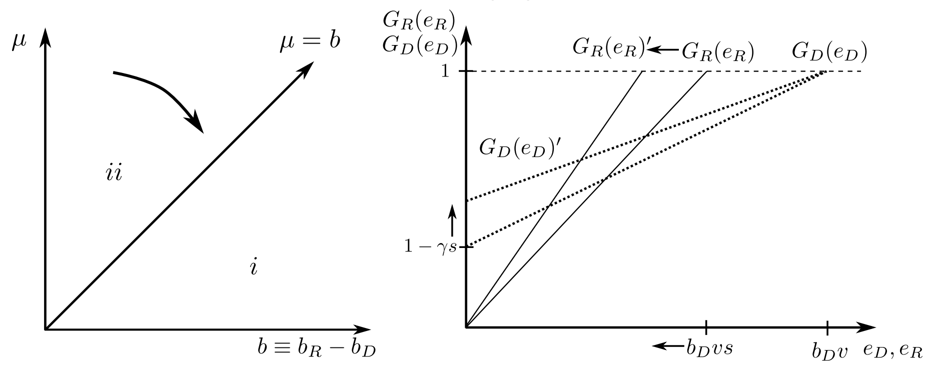
$$\int_0^{b_D v s} \int_0^{\frac{e_R}{s}} dG_D(e_D) dG_R(e_R) = 1 - \frac{1}{2}\gamma s \quad > \frac{1}{2} \tag{2.9}$$

$$\int_0^{b_D v} \int_0^{e_D s} dG_R(e_R) dG_D(e_D) = \frac{1}{2}\gamma s \quad < \frac{1}{2} \tag{2.10}$$

When $\mu = b$, the candidate P_R's size advantage equals the swing mass, the fraction $1 - \frac{1}{2}\gamma s = 1$, and the advantaged candidate wins with certainty. And, when b=0, the candidates' have the same size base, the fractions $s = \frac{\mu - b}{\mu + b}$ and $\gamma = \frac{b_D}{b_R}$ are both equal to 1, and the district is perfectly competitive. Both candidates have the same expenditure strategy, and both candidates have a $\frac{1}{2}$ chance of winning.

2.1.1. *Prediction Intuition.* Refer to Figure 2.3. Consider moving in an arc clockwise from the μ-axis ($b = 0$) toward the $\mu = b$ region-boundary. Along the vertical axis, district demographics favor neither candidate; the candidates have the same size base. The mixed Nash expenditure strategies are identical (neither candidate has a mass point at 0) and both mix uniformly up to their seat valuation. Also, both candidates have probability $= \frac{1}{2}$ of winning. As we move along the arc clockwise, meaning that the advantaged candidate's base advantage grows and the advertising-responsive swing mass decreases, the probability the disadvantaged candidate expends 0 (or concedes) increases and his probability of winning diminishes. The probability that advantaged candidate wins increases and the upper bound of the advantaged candidate's support diminishes. In fact, as the arc approaches the mu=b region-boundary (the mass of advertising-responsive voters diminishes), the model expenditure predictions converge pointwise to the Median Voter Model predictions; no expenditures and the advantaged candidate, who must have the median voter (otherwise could not be in region i), wins with probability 1.

FIGURE 2.3. Changing Variables



3. Testable Predictions and Estimation

Previous research by Hendricks & Porter (1988) and Baldwin et al. (1997) used ex-post realizations of object values to compare observed bid behavior with Nash behavior to test for collusive bid behavior. Alternatively, we use observed expenditure behavior to infer object valuation. Table 2 explicitly state some of the model predictions.[2] While previous work has relied on maximum likelihood (ML) techniques to estimate auction models, in this paper we estimate the value of a congressional seat using optimal General Methods of Moments (GMM), matching the empirical moments to the moment predictions. We do this because the model does not have an idiosyncratic error term. Instead, we use the model's predicted strategic

[2]There are many more moment predictions but these are the moments that are pertinant to estimating the value of a congressional seat.

uncertainty to estimate the valuation. The lack of error term has the benefit of not making any distributional assumptions about unobservables. The cost of the lack of an error term is that ML estimates of the valuation would be driven by the outlier expenditures. ML estimation matches every single distributional assumption of the model to estimate v, including the order statistics. As the Table 2 shows, v determines the support of the expenditure distributions and is therefore a parameter in the order statistics. Any extreme candidate expenditure would drive the ML estimate of v up to unreasonable levels.

To prevent this, we estimate the value of the congressional seat based on the first two predicted moments, the mean and the variance of expenditures. These moment predictions are used to estimate the valuation of a congressional house seat that would have given rise to the observed candidate expenditure behaviors. Again, this allows us to estimate the per partisan seat value v without introducing an error term. Since there is no strategic uncertainty in region i , we estimate v only with data from districts in region ii.

TABLE 2. Model Predictions in Region ii

Candidate	$p = D$	$p = R$
Moment Predictions		
$E(e_p \mid \mu, b_D, b_R)$	$\frac{1}{2}\gamma(b_D vs)$	$\frac{1}{2}b_D vs$
$Var(e_p \mid \mu, b_D, b_R)$	$(b_D v)^2(\frac{1}{3}\gamma s - \frac{1}{4}(\gamma s)^2)$	$\frac{1}{12}(b_D vs)^2$
Order Statistics		
$Min(e_p \mid \mu, b_D, b_R)$	0	0
$Max(e_p \mid \mu, b_D, b_R)$	$b_D vs$	$b_D v$

4. DATA

For this analysis, a unique data set of voter registration statistics and candidate expenditures was compiled. We compiled data on voter registration statistics by Congressional district from the years 2002, 2004, 2006, and 2008. We were forced to exclude some congressional districts for two main reasons. Not all states require voters to register for a political party when they register to vote. Generally, only the states with closed primary elections require and record the registered party for each voter. Also, some states keep voter registration statistics by county and their counties do not have the same borders as their districts. We use the voter registration statistics by Congressional as the measure of each candidates base voters, b_p, and therefore we do not include any districts where this measure is unavailable. We then define μ as the the number of registered voters who are not registered for either of the major parties.

Voters register with a party mainly so that they may participate in their party's primary. As was touted during the 2008 presidential primaries, it is possible that voters register with their less favored party so they can do damage to that party resulting in greater benefit for their favored party. In such a case, a strategic registrant would help elect the candidate from their less favored party that they feel has the greatest chance of losing to their favored party's candidate. Registrants employing such tactics are predicted to be a rare occurrence. Nonetheless, to mitigate any such strategic effect, we used registration statistics that were reported as close to the general election as possible.

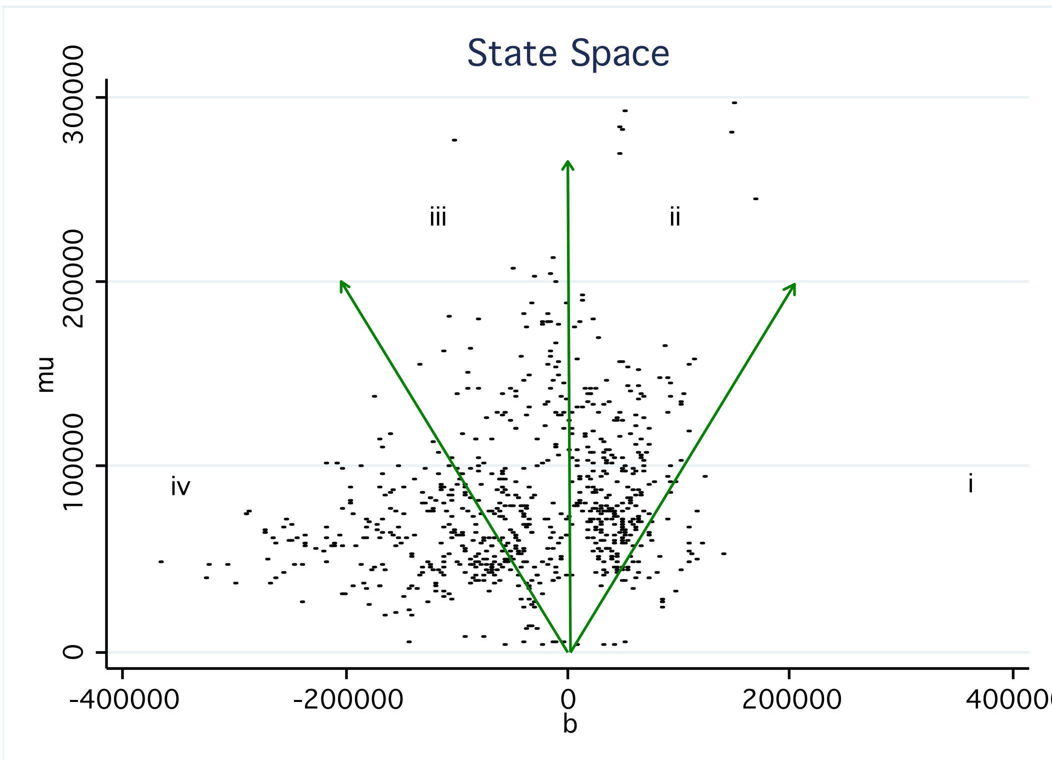


FIGURE 4.1. Congressional Districts in the (b,μ) State Space

The candidate campaign expenditure data comes from the United States Federal Election Commission (FEC) and all expenditures are converted to 2008 dollars. The FEC provides summary files of total election cycle expenditures for all candidates. Overall, we were able to collect voter registration statistics by party and link the candidate expenditures from 814 congressional district-election pairs. Our model does not incorporate any potential third party candidates and additional candidates have the potential to change the equilibrium in an all-pay auction. Therefore, we exclude any district where a third party candidate ran and spent more than \$10,000. We only include districts that have a Democratic facing a Republican, or a single Democrat or Republican running unopposed. This restriction excludes 65 districts. The remaining 749 districts are plotted in the (b, μ) state space in Figure 4.1. Recall that the theory was derived assuming that the Republican candidate was advantaged. This state space graph now shows all the districts. The Republican candidate is advantaged in regions *i* and *ii* and the Democrat is advantaged in regions *iii* and *iv*. Region *iii* is analogous to region *ii* in the that the model predicts mixed Nash expenditure strategies and region *iv* is analogous to region *i* in that the model predicts that the Republican candidate should concede to the Democratic candidate.

Figure 4.1 shows that there is very large variation in the strategic environment of these congressional districts.[3] Since we do not introduce an error term into our model, and estimate the value of a congressional seat solely off of the strategic uncertainty, we dismiss districts for which our model predicts concession.[4] While this is exclusion is necessary to estimate v from the model, the removal of the extreme regions *i* and *iv* should intuitively help provide a better estimate of the value of a congressional seat. The regions *ii* and *iii* are much less skewed to either party. The elections in these regions should be much more competitive and expenditures will matter more in the determination of the winner. Therefore, the expenditures should provide a better indication of the valuation of the congressional seat.

Table 3. Summary Statistics

Variable	Mean	Std. Dev	Min	Max
Registered Democrats (b_D)	140,724	33,532	33,953	280,575
Registered Republicans (b_R)	148,856	42,545	18,338	252,511
Swing Voters (μ)	100,052	45,347	5,097	296,248
Democratic Expenditure (e_D) ($)	911,372	990,843	0	5,591,222
Republican Expenditure (e_R) ($)	1,101,913	1,096,256	0	8,680,645
Districts		438		

Note: This table presents the summary statistics for all the districts in regions *ii* and *iii*.

The data used to estimate v consists of partisan voter registration statistics from the 438 congressional districts located in the competitive regions of the district state-space, regions *ii* and *iii*. The summary statistics are presented in Table 3. Even though Figure 4.1 shows large variation in the strategic state space,

[3]One striking feature about this graph is that there are numerous districts in region *iv*, where the Democrat is measured to be heavily advantaged. There is a surprising lack of balance in region *i*, where a Republican is heavily advantaged. This fact does not affect the results of the this paper, but could have implications about the partisanship of Congressional districts in state's with closed primaries.

[4]Districts where $b \geq \mu$.

these statistics show that the districts are very similar on average. There are only about 8,000 less registered Democrats on average and there is a sizable amount of swing voters. Additionally, these stats show that the Republican candidate outspends the Democrat by about 10% on average.

5. Estimation Results

This section presents the estimated v values, $\hat{v}$ and uses this estimate to calculate the distribution of congressional seat values. Table 4 presents three different estimates of v. The first estimates v using only one moment, the expected value of the advantaged candidate's expenditure. The second uses both expected values for the advantaged and the disadvantaged candidate. The third specification estimates v with both expected values and also both predicted variance moments. All estimates are done using optimally weighted GMM.

Table 4.

Estimated v Values

$\hat{v}$	44.797***	40.198***	29.996***
	(2.2552)	(2.0228)	(0.6678)
Observations	438	438	438
Objective Value	5.0e-17	0.0757	0.27979
Overid Stat		33.135	122.55
p =		0	0

Note: This table presents the estimated v values for three separate specifications. Column (1) uses only the mean advantaged expenditure moment. Column (2) uses both mean expenditure moments. Column (4) uses the both candidates mean and variance of expenditure moments. Optimally weighted standard errors are reported in parentheses and the asterisks indicate significance levels of the coefficients, *** p<0.01, ** p<0.05, * p<0.1.

All three estimates are highly significant. The value becomes smaller much and more precisely identified once the variance moments are included. This occurs because the variance moments incorporate much stronger model predictions than the mean predictions. The tradeoff is that the estimates are statistically over identified. However, this is expected given that we only have one parameter in the model.[5] Additionally, the estimates become statistically over identified as soon as one additional moment is included.

Therefore, we use the third estimate, $\hat{v} = 29.996$, to calculate the valuation of all the congressional seats. $\hat{v}$ is interpreted as the 2008 dollar value of the congressional seat to a partisan voter. The estimates are multiplied by the number of partisan voters to get the valuation to each candidate and these valuations are shown in Figure 5.1. These valuation vary greatly over the two candidates but the overall average of the valuations is about $4.5 million.

[5]Future work will parameterize the valuation to allow for more variation based on other district characteristics, such as the mean income level in the district.

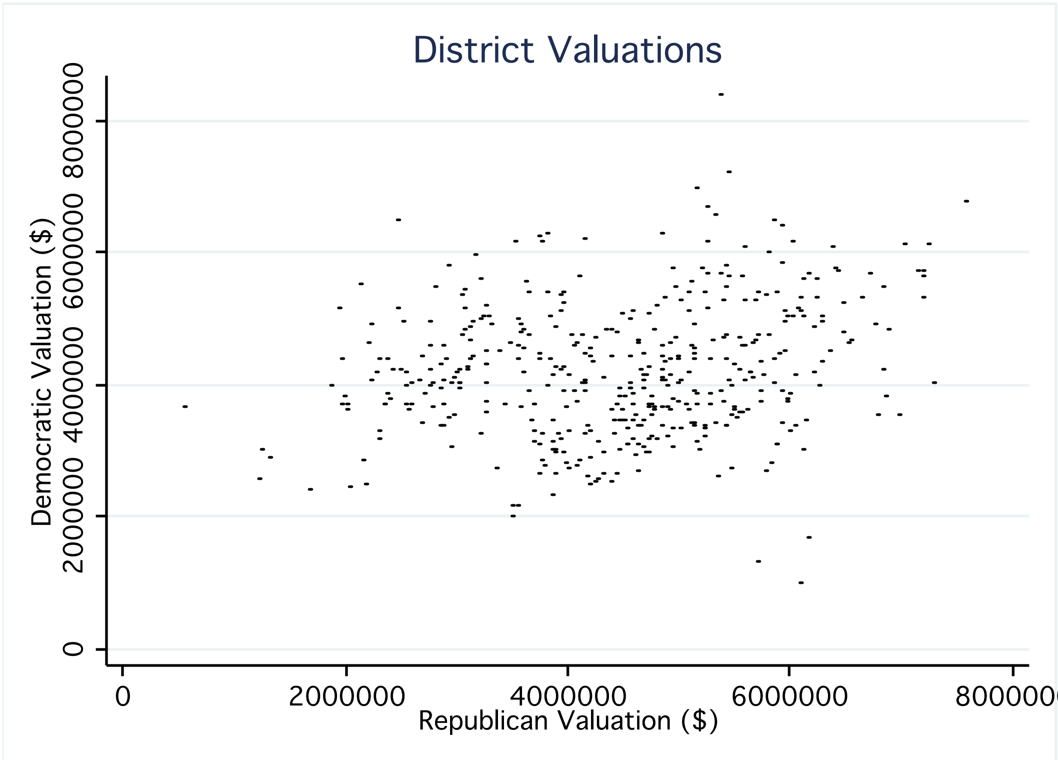


FIGURE 5.1. Valuations of Congressional Districts

6. CONCLUSION

This paper models Congressional campaign expenditures as bids in an asymmetric all-pay auction. The model predicts a wide variety of expenditure patterns and outcomes, even between districts that have similar demographics. Using the observed expenditure patterns and predictions from the model, we estimate that a congressional seat is worth about $30 per registered partisan and that on average the seat is worth about $4.5 dollars. In future research, the model and estimates can be used to analyze the effects of campaign finance reforms and redistricting policies. The change in optimal candidate expenditure patterns can be derived from the change in policies to see how candidates will react to the new policies and the effect that these change in expenditure patterns will have on election outcomes.

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